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## Math 238 Test # 2

No aids: Closed book. You are not permitted to consult with your fellow students in any way. Time: 80 minutes.

Question 1. (4 pts) Solve the initial value problem

$$y' = \frac{1}{y}, \quad y(0) = -1.$$

Question 2. (3 pts) Find a continuous solution to the initial value problem on  $[1, 3]$ :

$$y' + \frac{1}{t}y = \begin{cases} 3t, & 1 \leq t \leq 2 \\ 0, & 2 < t \leq 3 \end{cases} \quad y(1) = 1$$

Question 3. (2 pts) The initial value problem  $y' = \sqrt{|y|}$ ,  $y(0) = 0$  has two solutions

$$y \equiv 0 \text{ and } y^* = \begin{cases} t^2/4, & t \geq 0 \\ -t^2/4, & t < 0 \end{cases}$$

Why doesn't the existence and uniqueness theorem of Chapter 3 work in this case?

Question 4. (2 pts) Solve the Bernoulli differential equation  $y' + \frac{y}{t} = t^2 y^2$  by finding an appropriate substitution which linearizes the problem.

Question 5. (4 pts) Solve the exact differential equation

$$1 + y^2 + 2(t+1)yy' = 0, \quad y(0) = 1.$$

Question 6. (4 pts) A particle of mass  $m$  falls into a stream of air moving with constant velocity  $v_0$  and is carried along with it. The particle is acted on by a force  $k(v - v_0)^2$ . Determine the distance  $x$  travelled and the velocity  $v$  as a function of time  $t$  under the initial conditions  $x(0) = v(0) = 0$ .

Question 7. (3 pts) Show that  $\{y_1 = t^2, y_2 = t^3\}$  is a fundamental set of  $t^2 y'' - 4ty' + 6y = 0$  for all  $t > 0$  and all  $t < 0$ . Show that  $\{y_1 = t^2, y_2 = t^3\}$  is in fact a fundamental set for all  $-\infty < t < \infty$ , were we understand that any legitimate solution has to be smooth (all derivatives are continuous functions for any  $t$ ).

Compute the Wronskian  $W[y_1, y_2](t)$ . What is the value of  $W[y_1, y_2](0)$ ? Is this a contradiction?